Numerical methods for optimal quantization

Post-Doctoral Position
INRIA Bordeaux Sud-Ouest

Author of the post-doctoral research subject:
B. de Saporta and F. Dufour,
Equipe Projet INRIA: CQFD
saporta@math.u-bordeaux1.fr
dufour@math.u-bordeaux1.fr

Title of the post-doctoral research subject:
Numerical methods for optimal quantization.

Required Knowledge and background: PhD in applied probability, optimization or related area, skills in probability, optimization, numerical probability.

Keywords: Quantization, Numerical probability.

Duration: one year.

Scientific Research context:
Quantization techniques [4,5] consist in approximating a continuous state space random variable by a random variable taking only finitely many values such that the difference between the original random variable and its discrete approximation is minimal for some well-chosen norm. Quantization methods have been developed recently in numerical probability for numerical integration to solve nonlinear filtering or optimal stochastic control problems with applications in finance or signal processing [1,6-9]. In particular, this technique has been successfully used to solve optimal stopping and impulse control problems for various classes of stochastic processes such as stochastic differential equations, Markov chains, piecewise deterministic Markov processes [1,2,3]. Therefore, the development of numerical techniques for providing *(quasi)* optimal quantization is crucial.

Post-doctoral researcher work description:
Given a random variable X, a quantization algorithm should provide a finite grid G and the quantized approximation of X is then defined by the closest-neighbor projection of X onto G. One of the main challenges is to compute the optimal grid ensuring that the distance between X and its quantized approximation is minimal for a Lp norm. There already exist algorithms for such purposes such as Lloyd's Method and the competitive learning vector quantization procédure [1,5,9].
The candidate will study the existing methods by reviewing their advantages and drawbacks and develop new numerical methods for computing optimal grids.

References:
[1] Bally, V. and Pagès, G. (2003). A quantization algorithm for solving multi-dimensional discrete-time optimal stopping problems. *Bernoulli* **9** 1003–1049.

[2] Bally, V. , Pagès, G. and Printems, J . (2005). A quantization tree method for pricing and hedging multidimensional American options. *Math*. *Finance* **15** 119–168.

[3] de Saporta, B. , Dufour, F. and Gonzalez, K. (2010). Numerical method for optimal stopping of piecewise deterministic Markov processes, *Annals of Applied Probability*, **20** 1607-1637.

[4] Graf, S. and Luschgy, H. (2000) Foundations of Quantization for Probability Distributions, Lecture Notes in Math. 1730. Berlin: Springer-Verlag.

[5] Gray, R. and Neuhoff, D. (1998). Quantization, *IEEE Trans. On Information Theory* **44** 2325-2382.

[6] Pagès , G. (1998). A space quantization method for numerical integration. *J. Comput*. *Appl. Math*. **89** 1–38.

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[8] Pagès, G. , Pham, H. and Printems, J . (2004). An optimal Markovian quantization algorithm for multi-dimensional stochastic control problems. *Stoch. Dyn*. **4** 501–545.

[9] Pagès, G. , Pham, H. and Printems, J . (2004). Optimal quantization methods and applications to numerical problems in finance. In *Handbook of Computational and Numerical Methods in Finance* 253–297. Birkhäuser, Boston, MA.