Numerical methods for optimal control of piecewise deterministic Markov processes

Post-Doctoral Position
INRIA Bordeaux Sud-Ouest

Authors of the post-doctoral research subject:
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Title of the post-doctoral research subject:
Numerical methods for optimal control of piecewise deterministic Markov processes.

Required Knowledge and background: PhD in applied probability or related area, skills in continuous-time stochastic processes, Markov processes, optimal stochastic control, numerical probability.

Keywords: Markov processes. Optimal control. Numerical probability.

Duration: one year.

Scientific Research context:
In 1980, M.H.A. Davis [1] introduced in probability theory Piecewise Deterministic Markov Processes (PDMP) as a general class of models suitable for formulating optimization problems in queuing and inventory systems, maintenance-replacement models, investment scheduling and many other areas of operation research.
In the continuous-time context, stochastic control theory has from the numerical point of view, been mainly concerned with Stochastic Differential Equations (SDEs in short). From the practical and theoretical point of view, the numerical developments for this class of processes are extensive and largely complete. It capitalizes on the connection between SDEs and second order partial differential equations (PDEs in short) and the fact that the properties of the latter equations are very well understood. It is, however, hard to deny that the development of computational methods for the control of PDMPs has received little attention. One of the main reasons is that the role played by the familiar PDEs in the diffusion models is here played by certain systems of integro-differential equations for which there is not (and cannot be) a unified theory such as for PDEs as emphasized by M.H.A. Davis in his book. To the best of our knowledge, there is only one attempt to tackle this difficult problem by O.L.V. Costa and M.H.A. Davis. The originality of this project consists in studying this unexplored area.

Post-doctoral researcher work description:
An impulse control strategy consists in a sequence of single interventions introducing a jump of the process at some controller-specified stopping time and moving the process at that time

to some new point in the state space. The impulse control problem consists in choosing a strategy (if it exists) that minimizes the expected sum of discounted running and intervention costs up to infinity, and computing the optimal cost thus achieved. Many applied problems fall into this class, such as inventory problems in which a sequence of restocking decisions is made, or optimal maintenance of complex systems with components subject to failure and repair.
The candidate will develop numerical methods for impulse control of PDMP's. The proposed approach will be based on quantization techniques for the underlying Markov chain defined by the post jump location and inter-arrival time such as developed in [2]. Convergence results will be expected and numerical examples will be investigated.

References:
[1] M.H.A. Davis. *Markov Models and Optimization.* Chapman and Hall, London, 1993.
[2] B. de Saporta, F. Dufour, K. Gonzalez. *Numerical method for optimal stopping of piecewise deterministic Markov processes*, Annals of Applied Probability, 2010, vol. 20, no 5, p. 1607-1637.