Estimation of the jump rate of a PDMP

ANR-09-SEGI-004

Partial report of team CQFD on task 1.1.2

The aim of this task is to focus on the estimation of the jump rate of a PDMP. R. Azaïs, F. Dufour et A. Gégout-Petit are involved in this task.

1 Context

Piecewise-deterministic Markov processes (PDMP's) have been introduced by Davis as a general family of non-diffusion stochastic models, involving deterministic motion punctuated by random jumps at random times. In this thesis, we propose and analyze nonparametric estimation methods for both the features governing the randomness of such a process.

We focus on a nonparametric procedure for estimating the conditional distribution $f(\xi, t)$ of the inter-jumping time S_{n+1} at time t given the previous post-jump location $Z_n = \xi$, from only one observation of the process within a long time. Our approach is based on methods investigated in the manuscript for nonhomogeneous marked renewal processes and a generalization of the well-known multiplicative intensity model introduced by Aalen in the middle of the seventies. Our strategy for estimating the conditional probability density function $f(\xi, t)$ consists in the introduction of two functions $l(A, B_k, t)$ and $H(A, B_k, t)$, where (B_k) is a finite partition of the state space E of the PDMP. They provide a way to approximate the density of interest $f(\xi, t)$ for $\xi \in A$, in the sense that $f(\xi, t)$ is close to the quantity

$$\sum_{k=1}^{p} l(A, B_k, t) H(A, B_k, t).$$

Roughly speaking, we state that the quantity $l(A, B_k, t)$ may be seen as the jump rate from A to B_k at time t and under the invariant distribution ν of the post-jump locations Markov chain (Z_n). Furthermore, we establish that the function $H(A, B_k, t)$ is exactly the conditional probability $\mathbb{P}_\nu(S_1 > t, Z_1 \in B_k | Z_0 \in A)$. In this context, a natural estimator of $f(\xi, t)$ is given by

$$\widehat{f}_n(A, t) = \sum_{k=1}^{p} \widehat{l}_n(A, B_k, t)\, \widehat{p}_n(A, B_k, t),$$

where $\widehat{l}_n(A, B_k, t)$ estimates $l(A, B_k, t)$, while $\widehat{p}_n(A, B_k, t)$ is the empirical version of $H(A, B_k, t)$. We prove the uniform consistency of $\widehat{f}_n(A, t)$ under some conditions of ergodicity and classical assumptions of regularity on the main features of the PDMP.

We have submitted two papers in international journal on this task.

Références

[1] Azaïs, R., Dufour, F., and Gégout-Petit, A. Nonparametric estimation of the jump rate for non-homogeneous marked renewal processes. Submitted, arXiv :1202.2212.

[2] Azaïs, R., Dufour, F., and Gégout-Petit, A. Nonparametric estimation of the jump rate for piecewise-deterministic markov processes. Submitted, arXiv :1202.2211.