Preventive maintenance for a structure subject to corrosion

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Partial report of team CQFD and Astrium on task 3.1.3

The aim of this task is to propose a an application of our numerical optimal stopping procedure to an example of corrosion in collaboration with Astrium. The partners involved are INRIA CQFD and Astrium. People involved are B. de Saporta (CQFD), F. Dufour (CQFD) and C. Elegbede (Astrium), with the collaboration of H. Zhang (CQFD).

1 Context

A complex system is inherently sensitive to failures of its components. Therefore maintenance policies must be determined in order to maintain an acceptable operating condition. The optimization of maintenance is a very important problem in the analysis of complex systems. It determines when maintenance tasks should be performed on the system. These intervention dates should be chosen to optimize a cost function, that is to say, maximize a performance function or, similarly, to minimize a loss function. Moreover, this optimization must take into account the random nature of failures and random evolution and dynamics of the system. Theoretical study of the optimization of maintenance is also a crucial step in the process of optimization of conception and study of the life service of the system before the first maintenance.

An example of maintenance is considered here. It is related to an aluminum metallic structure subject to corrosion. This example was provided by Astrium. It concerns a small structure within a strategic ballistic missile. The missile is stored successively in a workshop, in a nuclear submarine missile launcher in operation or in the submarine in dry-dock. These various environments are more or less corrosive and the structure is inspected with a given periodicity. It is made to have potentially large storage durations. The requirement for security is very strong. The mechanical stress exerted on the structure depends in part on its thickness. A loss of thickness will cause an over-constraint and therefore increase a risk of rupture. It is thus crucial to control the evolution of the thickness of the structure over time, and to intervene before the failure.

The only maintenance operation considered here is the complete replacement of the structure. Partial repairs are not allowed. Mathematically, this problem of preventive maintenance corresponds to a stochastic optimal stopping problem as explained by example in the book of Aven and Jensen [1]. It is a difficult problem, because on the one hand, the structure spends random times in each environment, and on the other hand, the corrosiveness of each environment is also supposed to be random within a given range. In addition, the optimal maintenance date that is searched for should be adapted to the particular history of each structure, and not an average one. The predicted maintenance date should also be updated given the past history of the corrosion process.

2 Approach

This maintenance problem can be formulated as an optimal stopping problem for a piecewise-deterministic Markov process (PDMP). This class of problems has been studied from a theoretical point of view in [2]. PDMP's are a class of stochastic hybrid processes that have been introduced by Davis [3] in the 80's. These processes have two components: a Euclidean component that represents the physical system (e.g. temperature, pressure, thickness loss) and a discrete component that describes its regime of operation and/or its environment. Starting from a state $\boldsymbol{x}$ and mode m at the initial time, the process follows a deterministic trajectory given by the laws of physics until a jump time that can be either random (e.g. it corresponds to a component failure or a change of environment) or deterministic (when a magnitude reaches a certain physical threshold, for example the pressure reaches a critical value that triggers a valve). The process restarts from a new state and a new mode of operation, and so on. This defines a Markov process. Such processes can naturally take into account the dynamic and uncertain aspects of the evolution of the system. A subclass of these processes has been introduced by Devooght [4] for an application in the nuclear field. The general model has been introduced in dynamic reliability by Dutuit and Dufour [7].

The theoretical problem of optimal stopping for PDMP's is well understood, see e.g. Gugerli [8]. However, there are surprisingly few works in the literature presenting practical algorithms to compute the optimal cost and optimal stopping time. To our best knowledge only Costa and Davis [9] have presented an algorithm for calculating these quantities for PDMP's. Yet, as illustrated above, it is crucial to have an efficient numerical tool to compute the optimal maintenance time in practical cases. The objective of the present paper is to demonstrate the high practical power of the theoretical methodology described in [2]. Applying this general approach to a specific real-life industrial example brings new technical difficulties. More precisely, the algorithm given in this paper computes the optimal cost as well as a quasi optimal stopping rule, that is the date when the maintenance should be performed. As a byproduct of our procedure, the distribution of the optimal maintenance dates is also obtained, and dates such that the probability to perform a maintenance before this date is below a prescribed threshold can also be computed.

3 Results

This method has been implemented on an example of optimization of the maintenance of a metallic structure subject to corrosion, and very satisfactory results were obtained, very close to theoretical values, despite the relatively large size of the problem. These results are interesting for Astrium in the design phase of the structure to maximize margins from the specifications and to consolidate the available dimensional margins. Thus, tools are proposed to justify that with a given probability no maintenance will be required before the end of the contract.

4 Dissemination of results

This work was presented in a conference [5] and published in an international peer-reviewed journal [6].

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