Given the points of regime change, estimation of parameters of the current regime by filtering of the PDMP, properties of the estimators.

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Partail report of team CQFD on task 1.1.2

This task is devoted to the development of theoretical and numerical tools for the parameter estimation of PDMP's in a general context. People involved are R. Azaïs (CQFD), F. Dufour (CQFD) and A. Gégout-Petit (CQFD).

1 Context

Piecewise-deterministic Markov processes (PDMP's) have been introduced in the literature by M.H.A. Davis [3] as a general class of stochastic models. PDMP's are a family of Markov processes involving deterministic motion punctuated by random jumps. The motion of the PDMP $\{X(t)\}$ depends on three local characteristics, namely the flow ϕ, the jump rate λ and the transition measure Q, which specifies the post-jump location. Starting from x the motion of the process follows the flow $\phi(x,t)$ until the first jump time T_1 which occurs either spontaneously in a Poisson-like fashion with rate $\lambda(\phi(x,t))$ or when the flow $\phi(x,t)$ hits the boundary of the state-space. In either case the location of the process at the jump time T_1: $X(T_1) = Z_1$ is selected by the transition measure $Q(\phi(x,T_1),\cdot)$. Starting from Z_1, we now select the next interjump time $T_2 - T_1$ and postjump location $X(T_2) = Z_2$. This gives a piecewise deterministic trajectory for $\{X(t)\}$ with jump times $\{T_k\}$ and post jump locations $\{Z_k\}$ which follows the flow ϕ between two jumps. We denote $S_k = T_k - T_{k-1}$ for $k \geq 1$.

In this work, we suppose the flow given by physics laws and we want to make some inference on λ. ϕ being deterministic, the problem can be rewritten as a problem of estimation of the rate $\lambda(z,t)$ with $z \in E$ with E an open set of a separable metric space. We have an ergodicity assumption on the observed PDMP and the asymptotic is in the time of observation of the process.

2 Results

We distinguish three cases :

1. E is finite. In this case, we easily estimate each of the cumulated risk functions $\Lambda(z,t) = \exp(-\int_0^t \lambda(z,s)ds)$ corresponding to each of $z \in E$ by a Nelson Aalen estimator. The results is based on the decomposition in semi-martingale of the following counting process in an appropriate filtration:
$$\forall t \geq 0,\ N_n(z,t) = \sum_{i=0}^{n-1} \mathbf{1}_{\{S_{i+1} \leq t\}} \mathbf{1}_{\{Z_i = z\}},$$
We obtain the estimator of the rate $\lambda(z,t)$ by smoothing of the estimator of Λ.

2. E is an open set of a general separable metric space but the transition measure Q does not depend on the time spent in the current regime. In this case, we suppose the rate $\lambda(z,t)$ Lipschitz and the process ergodic with a stationary law denoted by ν. We first construct an estimation of the cumulated rate knowing that z belongs to a set A such that $\nu(A) > 0$ by :
$$\widehat{L}_n(A,t) = \sum_{i=0}^{n-1} \frac{1}{Y_n(A,S_{i+1})} \mathbf{1}_{\{S_{i+1} \leq t\}} \mathbf{1}_{\{Z_i \in A\}} \qquad \text{with} \qquad Y_n(A,t) = \sum_{i=0}^{n-1} \mathbf{1}_{\{S_{i+1} \geq t\}} \mathbf{1}_{\{Z_i \in A\}}.$$
We show the consistence of the estimator. Smoothing $\widehat{L}_n(A,t)$ and using a fine partition of E allow us to obtain an uniform result for the approximation of the rate $\lambda(z,t)$, in some sense in t and z.

3. E is an open set of a general separable metric space and the transition measure Q depends on the time spent in the current regime. Here, we loose some conditional independence between the S_i's and the whole set of the locations of the jump $\{Z_1, \ldots, Z_n\}$. We have to make a detour for the estimation of the law of the time S_{k+1} knowing the current Z_k by the the law S_{k+1} knowing (Z_k, Z_{k+1}). The method gives an estimation of the conditional density of S_{k+1} given Z_k.

We have made simulation studies that give expected results. A R package for this estimation method is in progress.

3 Dissemination of results

This work is a part of the PhD Thesis of R. Azaïs founded by the ANR. R. Azaïs presented its at "Rencontres des Jeunes Statisticiens" in 2011 September [2]. The work will be soon submitted to a international peer-reviewed journal for publication.

References

Publications of the Fautocoes team

[1] Azaïs, R., Dufour, F., Gégout-Petit, A. Estimation of the jump rate of a PDMP. *In preparation.*

[2] Azaïs, R. Estimation du taux de saut pour une classe de processus markoviens de saut,. *Rencontres des Jeunes Statisticiens*, Aussois, 2011.

References in the text

[3] Davis, M. H. A. *Markov models and optimization*, vol. 49 of *Monographs on Statistics and Applied Probability.* Chapman & Hall, London, 1993.