Estimation of the characteristics of a PDMP

ANR-09-SEGI-004

Partial report of team CQFD on task 1.1.2

The aim of this task is to focus on the estimation of the kernel of a PDMP. R. Azaïs, F. Dufour et A. Gégout-Petit are involved in this task.

1 Context

Piecewise-deterministic Markov processes (PDMP's) have been introduced by Davis as a general family of non-diffusion stochastic models, involving deterministic motion punctuated by random jumps at random times. In this thesis, we propose and analyze nonparametric estimation methods for both the features governing the randomness of such a process.

We assume that the transition kernel $Q(x, dy)$ of the PDMP admits a density $q(x, y)$ with respect to the Lebesgue measure. We propose a nonparametric recursive estimator $\widehat{q}_n(x, y)$ for the transition density $q(x, y)$, under some assumptions of ergodicity on the embedded Markov chain of the PDMP. The recursive estimator of $q(x, y)$ that we consider may be written as follows,

$$\widehat{q}_n(x,y) = \frac{\displaystyle\sum_{j=1}^{n+1} \frac{1}{w_j^{2d}} K\left(\frac{\Phi_{Z_{j-1}}(S_j) - x}{w_j}\right) K\left(\frac{Z_j - y}{w_j}\right)}{\displaystyle\sum_{j=1}^{n+1} \frac{1}{v_j^{d}} K\left(\frac{\Phi_{Z_{j-1}}(S_j) - x}{v_j}\right)},$$

where $\Phi_x(t)$ is the deterministic flow of the PDMP. In addition, K is a kernel function from $\mathbb{R}^d$ to $\mathbb{R}_+$ satisfying a few conditions. In this work, we establish the pointwise convergence of $\widehat{q}_n(x, y)$. Furthermore, we prove that this estimator satisfies a central limit theorem with rate $n^{(1-\alpha d)/2}$ under some conditions on the bandwidth parameters α and β and a Lipschitz mixing assumption. Finally, numerical simulations illustrate our theoretical results for both the estimation procedures.

R. Azaïs defended is PhD on this subject in July, 2013.

Références

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[2] Azaïs, R., Dufour, F., and Gégout-Petit, A. Nonparametric estimation of the jump rate for piecewise-deterministic markov processes. In revision, arXiv :1202.2211.

[3] Azaïs, R. A recursive nonparametric estimator for the transition kernel of a piecewise-deterministic Markov process Accepted for publication in ESAIM : Probability and Statistics (2013). arXiv :1211.5579v2

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